**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Time Left = 60-10 = 50 minutes

P(not meeting the commitment) = 1 – P(meeting the commitment) = 1 –stats.norm.cdf (50,45,8) = 0.2659

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

P(X>44) = 1 – P(X<=44) = 1 –stats.norm.cdf (44,38,6) = 0.158 = 15%

P(38<X<44) = P(X<44) – P(X<38) = 0.341 = 34%

Hence P(X>44) < P(38<X<44)

Given statement is FALSE

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

P(X<30) = stats.norm.cdf(30,38,6) = 0.0912 = 9.12%

9.12% of 400 employees = 36 (approx.)

Hence given statement is TRUE

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

E(2X1) = 2 E(X1)

* Distribution of 2X1 is also Normal

Parameters:

Var(2X1) = 2 Var(X1)

SD(2X1) = 2 SD(X1)

E(X1+X2) = E(X1) + E(X2)

Sum of 2 normal distributions also follows normal distribution

Var(X1+X2) = Var(X1) + Var(X2)

SD(X1+X2) =

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Mean = 100, SD = 20

Interval Estimate = Point Estimate ± Margin of Error

* ± Zσ

Confidence Interval: ± Zσ

Z @ 99% = stats.norm.ppf(0.995) = 2.57

CI : [100-2.57(20/sqrt(1)), 100+2.57(20/sqrt(1))]

[48.6, 151.4]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

E(aX1) = a E(X1)

E(X1+X2) = E(X1) + E(X2)

E(X1) = 5, E(X2) = 7

E(company) = E(Profit1 + Profit2) in $ Million

= E[45\*(Profit1 + Profit2)] Million Rupees

= 45\*(5+7) Million Rupees

= 540 Million Rupees

SD(company) = SD(Profit1 + Profit2) in $ Million

= 45\* Million Rupees

= 45\* Million Rupees

= 45\*5 Million Rupees = 225 Million Rupees

Company ~ N (540, 2252)

Z@95% = 1.96

CI: Mean±Z\*SD = 540 ± 1.96\*225 = (99, 981)

1. Specify the 5th percentile of profit (in Rupees) for the company

5th percentile => 90% Confidence Interval

Interval Estimate = Point Estimate ± Margin of Error

* ± Zσ
* 1.64\*
* \* 45
* (171, 909)

1. Which of the two divisions has a larger probability of making a loss in a given year?

Loss => Profit = 0

1. P(0,5,3) = 0.0477 = 4.77%
2. P(0,7,4) = 0.04 = 4%

P(0,5,) > P(0.7,4)